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New methodologies for the determination of yield pressure of marine clays based on CPTU and geological history

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Articles

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Abstract

This study investigates the use of empirical formulas for the determination of the yield pressure of marine clays through the CPTU. One such formula is that of Kulhawy & Mayne, which has previously been analyzed by the author (Massad, 2010), who proposed a methodology for determining its empirical factor for soils with known stress history and with cone resistance (q_t) varying linearly with depth. These conditions have been observed in marine clays in Brazil and abroad. In the present work the author extends this methodology to two other formulas of Mayne (2017), which make use of cone resistance (q_t), pore water pressure (u_2) and hydrostatic pressure (u_o), adjusting the empirical factors for cases of linear relationships of q_t and u_2 with depth. An analysis is conducted to identify the conditions necessary for these formulas to consistently produce the same yield pressure. The effectiveness of the proposed approach is demonstrated through three case studies, which include two European marine clays and one Brazilian marine clay, supported by CPTU data, specific unit weights, oedometer tests, and geological history.

1. Introduction

Piezocone tests (CPTU) have been used to determine the yield or the preconsolidation pressure (σ'_a) of marine clays in Brazil and around the world. This is done with few mathematical models and many empirical correlations. However, it is important to consider the geological and geotechnical contexts when establishing such correlations for specific soils, as emphasized by Demers & Lerouiel (2002) and reiterated recently by Mayne (2017).

One of the most used empirical correlations is that of Kulhawy & Mayne, quoted by Coutinho et al. (2000), given by the equation:

$$\sigma'_a = \frac{q_t - \sigma_{vo}}{N_{\sigma t}} \tag{1}$$

The parameter q_t denotes the corrected cone tip resistance, while σ_{vo} represents the total vertical stress. The empirical factor $N_{\sigma t}$ is typically taken as 3.3, proposed by Mayne et al. (1998), or alternatively 3.4, suggested by Demers & Lerouiel (2002).

Additionally, Mayne (2017) introduced two other expressions applicable to inorganic clays exhibiting low sensitivity, as outlined below:

$$\sigma'_a = k_2 \cdot (q_t - u_2) \tag{2}$$

$$\sigma'_a = k_3 \cdot (u_2 - u_o) \tag{3}$$

where u_o and u_2 denote the hydrostatic pressure and the pore pressure measured at the base of the cone, respectively, and k_2 and k_3 are empirical factors. Mayne (2017) recommended the following values for these empirical factors: $N_{\sigma t}$=3.0, k_2=0.60 and k_3=0.54. For this reason, Equations 1, 2 and 3 will be referred to hereafter as the 3 Mayne's Formulas. In addition, he emphasized that these factors may require adjustments to account for the specific properties of each geomaterial.

2. Parallelism between σ'_a and σ'_{vo} and its slight deviation

Based on the geological history of some Brazilian marine clays, Massad (2009) showed that there is a parallelism between yield pressure (σ'_a) and the effective vertical initial pressure (σ'_{vo}), that is:

$$\sigma'_a = \sigma'_{vo} + \Delta p \tag{4-a}$$

where Δp is constant due to overloads from: a) negative oscillations of the sea water level along time; b) sediment erosion; and d) dune weight.

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This parallelism may experience slight deviations under the phenomenon known as "aging", a form of creep deformation, reflecting time-dependent changes in soil structure under sustained loads over extended periods of time. When aging occurs, it is necessary to introduce an additional term, denoted as r, into the Equation 4-a, as follows:

$$\sigma'_a = r \cdot \left(\Delta p + \sigma'_{vo}\right) \tag{4-b}$$

Massad (2009) proposed the following equation using expression of Mesri & Choi (1979):

$$r = \frac{\sigma'_a}{\sigma'_{vo} + \Delta p} = \left(\frac{t}{t_p}\right)^{\frac{C_{\alpha e}/C_c}{1 - C_r/C_c}} \tag{4-c}$$

where C_{ae} is the secondary consolidation coefficient; C_c, the soil compression index; C_r, the recompression index; t_p, the time required for the development of primary consolidation; and t, the time of aging. The value of r usually ranges from 1 (indicating no aging effect) to values as high as 1.4, depending on the extent and duration of aging processes. When only aging is considered (Δp=0), r is equivalent to the over-consolidation ratio (*OCR*), meaning the yield pressure is directly proportional to the initial effective stress and *OCR* remains constant with depth (Massad, 2009). Incorporating the aging parameter r into predictive models is essential for accurately assessing the yield pressure of marine clays.

3. Linear relation between q_t and u_2 with depth

In addition, it has been found that both q_t and u_2 may vary linearly with depth (z) in some marine clays, that is:

$$q_t = a + b.z \tag{5}$$

$$u_2 = c + d.z \tag{6}$$

where a, b, c and d are constants. When the water table is located at a depth z_{wl}, the hydrostatic pressure (u_o) may be described as:

$$u_0 = \gamma_o \cdot \left(z - z_{wl}\right) \; for\; z \geq z_{wl} \; otherwise \; u_0 = 0 \tag{7}$$

with γ_o representing the unit weight of water. These linear relationships, integrated with geological and geotechnical history through Equation 4-b, provide a practical framework for applying the empirical Equations 1, 2 and 3 to estimate yield pressure.

4. Determination of the empirical factors of the 3 Mayne's Formulas

The empirical factors of Mayne's Formulas can be determined based primarily on the Equations 4-b, 5 and 6.

4.1 Determination of the factor $N_{\sigma t}$ of Equation 1

If the specific unit weight (γ_n) is constant along depth (z), the total and the effective initial stresses can be written:

$$\sigma_{vo} = \sigma_{vo}^{z=0} + \gamma_n.z \tag{8}$$

$$\sigma'_{vo} = \sigma_{vo}^{'z=0} + \gamma'.z \tag{9}$$

where γ' is the submerged unit weight. If γ_n is not constant, it should be replaced at a depth z by $\overline{\gamma}_n$, the weighted average, given by:

$$\overline{\gamma}_n = \frac{\int_0^z \gamma_n.dz}{z} \tag{10}$$

Substituting Equation 1 into Equation 4-b and considering Equations 5, 8 and 9 the following expression may be derived, after some transformations:

$$N_{\sigma t} = \frac{b - \gamma_n}{r.\gamma'} \tag{11}$$

This equation was presented earlier by Massad (2009 and 2010).

4.2 Determination of the factor k_2 of Equation 2

Similarly, substituting Equation 2 into Equation 4-b and bearing in mind Equations 5, 6 and 9, the following expression results, after some transformations:

$$k_2 = \frac{r.\gamma'}{\left(b - d\right)} \tag{12}$$

4.3 Determination of factor k_3 of Equation 3

Finally, replacing Equation 3 into Equation 4-b and in view of Equations 6, 7 and 9, the following expression may be written, after some transformations:

$$k_3 = \frac{r.\gamma'}{\left(d - \gamma_o\right)} \tag{13}$$

4.4 Summary of the results so far

Table 1 summarizes the formulas of the 3 empirical factors.

Table 1. The empirical factors of Equations 1, 2 and 3.

Equation 1	Equation 2	Equation 3
$N_{\sigma t} = \frac{b - \gamma_n}{r.\gamma'}$	$k_2 = \frac{r.\gamma'}{\left(b - d\right)}$	$k_3 = \frac{r.\gamma'}{\left(d - \gamma_o\right)}$

Legend: See the attached list of symbols.

5. Condition for consistency among the 3 Mayne's Formulas

For Equations 1, 2 and 3 to return identical values for the yield pressure, the following conditions must be satisfied:

$$\sigma'_a = \frac{q_t - \sigma_{vo}}{N_{\sigma t}} = k_2 \cdot (q_t - u_2) = k_3 \cdot (u_2 - u_o) \tag{14}$$

Based on Equations 11, 12, and 13, these conditions can be reformulated using auxiliary variables t_1, t_2 and t_3:

$$t_1 = \frac{q_t - \sigma_{vo}}{q_t - u_2} = k_2.N_{\sigma t} = \frac{b - \gamma_n}{b - d} \tag{15}$$

$$t_2 = \frac{q_t - \sigma_{vo}}{u_2 - u_o} = k_3.N_{\sigma t} = \frac{b - \gamma_n}{d - \gamma_o} \tag{16}$$

$$t_3 = \frac{q_t - u_2}{(u_2 - u_o)} = \frac{k_3}{k_2} = \frac{b - d}{d - \gamma_o} \tag{17}$$

Note that t_1 and t_2 are functions of the natural unit weight γ_n while t_3 is independent of it. Notably, t_2 is equal to the inverse of the pore pressure parameter (B_q). If b, d and γ_n are constants, then, t_1, t_2 and t_3 also remain constant with depth, and the following matrix equation can be established:

$$\begin{bmatrix} t_1 - 1 & -t_1 & 1 \\ 1 & -t_2 & -1 \\ 1 & -(t_3 + 1) & 0 \end{bmatrix} . \begin{bmatrix} b \\ d \\ \gamma_n \end{bmatrix} = \begin{bmatrix} 0 \\ -\gamma_o.t_2 \\ -\gamma_o.t_3 \end{bmatrix} \tag{18-a}$$

This system may be written:

$$T.U = N \tag{18-b}$$

where T is the known 3 x 3 matrix; U is the vector of unknowns and N is the known vector. The determinant of T is null. In fact, applying Sarrus' Rule (Khattar, 2010) and considering $t_1.t_3 = t_2$, results:

$$Det(T) = [0 + t_1 - (t_3 + 1)] - [-t_2 + (t_1 - 1) * (t_3 + 1)] = 0 \tag{19}$$

This is a necessary condition to obtain the same σ'_a by the 3 Mayne's Formulas.

In practice, however, this system of Equations 18-a is ill conditioned due to measurement inaccuracies in the CPTUs and rounding errors in the calculations, which can result in unrealistic solutions.

To address this, Equations 18-a can be transformed into:

$$d = \frac{b + \gamma_o.t_3}{(1 + t_3)} \tag{20}$$

$$\gamma_n = \gamma_o + (b - \gamma_o), \left(1 - \frac{t_2}{1 + t_3}\right) \tag{21}$$

If b, d and γ_n do not satisfy these equations, then the consistency condition given by Equation 14 is not met, and the three Mayne's Formulas will return different values for yield pressure, namely: σ'_{1a}, σ'_{2a} and σ'_{3a} from Equations 1, 2 and 3, respectively.

To further analyze inconsistencies, auxiliary variables t'_1, t'_2 and t'_3 are defined as:

$$t'_1 = k_2.N_{\sigma t} = \frac{b - \gamma_n}{b - d} \equiv / t_1 = \frac{q_t - \sigma_{vo}}{q_t - u_2} \tag{22}$$

$$t'_2 = k_3.N_{\sigma t} = \frac{b - \gamma_n}{d - \gamma_o} \equiv / t_2 = \frac{q_t - \sigma_{vo}}{u_2 - u_o} \tag{23}$$

$$t'_3 = \frac{k_3}{k_2} = \frac{b - d}{d - \gamma_o} \equiv / t_3 = \frac{q_t - u_2}{(u_2 - u_o)} \tag{24}$$

where the symbol ≡/ means t'_i "not always equal to" the corresponding t_i values (i=1, 2 and 3). The differences between the t_i and the t'_i mean that the condition for identical yield pressure σ'_a is not satisfied. Using Equations 1, 2 and 3, combined with Equations 22, 23 and 24, the deviations among the calculated yield pressures can be quantified as follows:

$$\frac{\sigma'_{a1} - \sigma'_{a2}}{\sigma'_{a2}} = \frac{t_1 - t'_1}{t'_1} \tag{25}$$

$$\frac{\sigma'_{a1} - \sigma'_{a3}}{\sigma'_{a3}} = \frac{t_2 - t'_2}{t'_2} \tag{26}$$

$$\frac{\sigma'_{a2} - \sigma'_{a3}}{\sigma'_{a3}} = \frac{t_3 - t'_3}{t'_3} \tag{27}$$

It is easy to see that if $t_1 \neq t'_1$, then at least one of the $t_i \neq t'_i$ for i=2 or 3.

Another way to deal with the ill conditioning of the system of Equations 18-a is to check whether the solution is consistent with measured natural unit weight (γ_n) or yield pressure (σ'_a), when available, as well as with the site's geological knowledge.

6. Approach to determine σ'_a

To solve the ill conditioned system of Equations 18-a it is assumed that b is known precisely, which only depends on the measurement of q_t with the depth (z), assumed to be linear. Thus, Equations 20 and 21 allow to determine d and γ_n.

The value of d, in turn, depends on the measurement of u_2 with the depth (z), assumed also to be linear, and may be used to validate the calculations. As mentioned before, t_2 in Equation 21 depends on σ_{vo}, and then on γ_n, implying in iterative calculations, which may be done as follows:

- A value for soil unit weight γ_{n1} is assumed.
- The values of t_1, t_2 and t_3 are determined as a function of z, based on Equations 15, 16 and 17, expressed in terms of the parameters q_t, u_2, σ_{vo} and u_o.
- It is verified whether the values of t_1, t_2 and t_3 are practically constant with depth (z); the mean values of these parameters are determined.
- Enter the value of b into the Equations 20 and 21 and determine the γ_{n2} and d;
- If $\gamma_{n2} \neq \gamma_{n1}$ the calculations are repeated until convergence.
- The computed parameter d should be compared with the value obtained directly from the graph $u_2=f(z)$ to validate the mathematical solution of the system of equations, which is not necessarily the actual response, as explained above.

7. Constancy and consistency of B_q with depth

One consequence of Equations 14, which imply the same value of the yield pressure, is the constancy of B_q, the Pore Pressure Coefficient, with depth (z).

In fact, from:

$$B_q = \frac{u_2 - u_o}{q_t - \sigma_{vo}} = \frac{1}{t_2} \tag{28}$$

it follows:

$$\frac{dB_q}{dz} = \frac{\left(q_t - \sigma_{vo}\right).\left(\frac{du_2}{dz} - \gamma_o\right) - \left(u_2 - u_o\right).\left(\frac{dq_t}{dz} - \gamma_n\right)}{\left(q_t - \sigma_{vo}\right)^2} \tag{29-a}$$

Or, considering Equations 5, 6 and 16:

$$\frac{dB_q}{dz} = \frac{\left(q_t - \sigma_{vo}\right).\left(d - \gamma_o\right) - \left(u_2 - u_o\right).\left(b - \gamma_n\right)}{\left(q_t - \sigma_{vo}\right)^2} = 0 \tag{29-b}$$

That is, B_q is constant with z; small variations may occur, denoting deviations in the values of σ'_{a1}, σ'_{a2} and σ'_{a3}. Another way to prove the assertion is to consider that t_2 in Equation 28 is constant along the depth(z).

8. Three case histories

Three cases of marine clays will be analyzed, namely: a) Bothkennar Clay (UK); b) Torp Clay (Sweden) and Jurubatuba Clay (Santos, Brazil).

8.1 Bothkennar clay (UK)

The silty soft clay of Bothkennar is located on the banks of the River Forth between Edinburgh and Glasgow (UK). It has been the subject of many studies over the years, due to its homogeneity and uniformity. In 1992, volume XLII, no. 2 of the Geotéchnique Journal, was dedicated to it and, more recently, it was addressed by Mayne (2016) and Ferreira (2021).

According to Nash et al. (1992b), Bothkennar Clay is overconsolidated due to processes that occurred after its deposition between 8,500- and 6,000-years BP, including erosion, sea level fluctuation and surface drying, but that aging would have been its main cause. These authors presented three sets of consolidation tests (oedometer), which were entitled as: “First set (piston)”, “Large increment” and “Laval”. These titles indicate the sampler used to collect the undisturbed samples or how the tests were performed. Only the first and third sets will be considered next.

Based on these tests and on settlement observation data, Ferreira & Massad (2022a, b) assigned the value 1.33 to parameter r of Equation 4-c. In fact, data from Nash et al. (1992a, b) reveals $C_{\alpha e}/C_c$=0.04; C_r/C_c=0.1; t_p=10 years, then for an aging time t=6,000 years, r=(6,000/10)$^{0.04/(1-0.1)}$=1.33 from Equation 4-c. For t=8,500 years r=1.35.

Mayne (2016) analyzed the results of two CPTUs, presented by Nash et al. (1992b) and Powell & Lunne (2005); one of them was used to produce Figure 1. The water level was at a depth of 0.8 m and the natural specific unit weight of the clay (γ_n) was 16.7 kN/m^3.

Below the desiccated crust, about 3.5 m thick, the values of q_t and u_2 vary linearly with depth, as shown in Figure 1, so the 3 Mayne’s Formulas, i.e., Equations 1, 2 and 3, can be used, with the corresponding empirical factors summarized in Table 1. The values of the empirical factors ($N_{\sigma t}$, k_2, k_3) and of the $\sigma'_{a,}$ are presented in Table 2 and in Figure 2a, respectively.

The parameters t'_1, t'_2 and t'_3 were determined using Equations 22, 23 and 24, with the values of b and d from Figures 1a and 1b respectively, and γ_n=16.7 kN/m^3. They are displayed in Table 3 and inserted into Figure 3 (dashed lines). This figure shows the variations of t_1, t_2 and t_3 with depth, using the same Equations 22, 23 and 24 and the values of q_t, σ_{vo}, u_2 and u_o, from Figure 2.

The average values of t_1/t'_1, t_2/t'_2 and t_3/t'_3 are 1.04, 1.01 and 0.97, respectively, which means, from Equations 25, 26 and 27, small deviations of 4% between σ'_{a1} and σ'_{a2}; 1% between σ'_{a1} and σ'_{a3} and -3% between σ'_{a2} and σ'_{a3}.

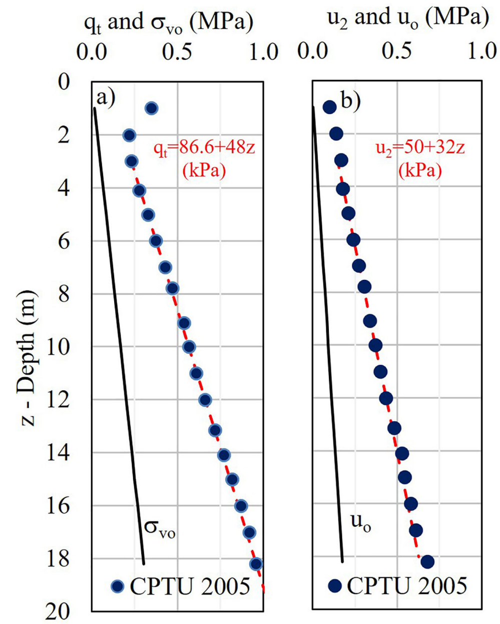

Figure 1. CPTU in Bothkennar – Prepared with data from Mayne (2016): a) q_t and σ_{vo}; and b) u_2 and u_o.

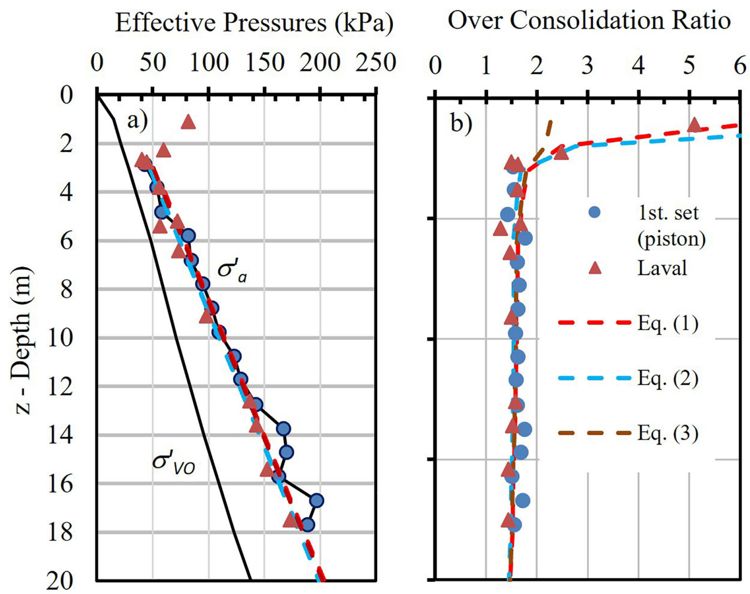

Figure 2. a) Effective Pressures σ'_{vo} and σ'_a; and b) OCR – Bothkennar Clay (UK).

Table 2. Results obtained for the 3 empirical factors of Equations 1, 2 and 3 - Bothkennar Clay.

Item	$N_{\sigma t}$	k_2	k_3
Mayne (2017)	3.0	0.60	0.54
Calculated	3.5	0.56	0.41

Table 3. Values of t'_1, t'_2 and 't_3 obtained through Equations 22, 23 and 24 as a function of b, d and γ_n.

Item	t'_1	t'_2	t'_3
Bothkennar Clay	1.96	1.42	0.73

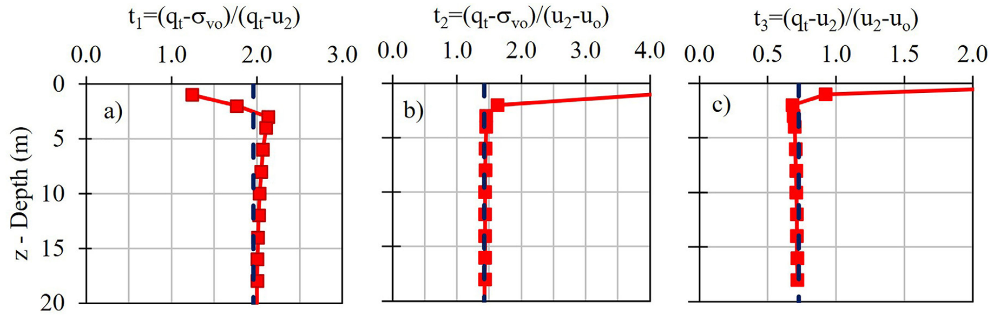

Figure 3. Values of: a) t_1; b) t_2 and c) t_3 varying with depth. The dashed lines represent t'_1, t'_2 and t'_3 – Bothkennar Clay (UK).

Supposing that γ_n was not available, the attempt to directly solve Equations 18-a would lead to the following terms of Equations 18-b:

$$T = \begin{bmatrix} 1.206 & -2.206 & 1 \\ 1 & -1.565 & -1 \\ 1 & 1.707 & 0 \end{bmatrix},$$

$$U = \begin{bmatrix} 48.0 \\ 32.2 \\ 13.2 \end{bmatrix} and \tag{30}$$

$$N = \begin{bmatrix} 0.00 \\ -15.65 \\ -7.07 \end{bmatrix}$$

Note that multiplying T by U results a vector whose transpose is [0,00 -15.65 -7.05], which is practically equal to N. Nevertheless, this solution gave γ_n =13.2 kN/m^3 (third element of U), very different from the measured value of 16.7 kN/m^3 and so was discarded. The corresponding values of yield pressures were also unrealistic. These results occur due to the ill conditioning of the system (18-a): the determinant of matrix T is 4.9E-03, almost zero.

From the analysis of these results, it can be concluded that:

- The σ'_a calculated through the 3 Mayne's Formulas, i.e., Equations 1, 2 and 3, with the empirical factors of Table 2, are very close to the values obtained through the consolidation tests, indicated in Figure 2a; below the desiccated crust, the over consolidation ratio (*OCR,* Figure 2b) vary roughly between 1.8 and 1.5, averaging around 1.6.
- The empirical factors calculated for Bothkennar Clay (Table 2) differ from those indicated by Mayne (2017). The differences are +17% for $N_{\sigma t}$, -7% for k_2 and -24% for k_3.
- Reporting again to Figure 2a and taking as reference the σ'_{a1} from Equation 1, the corresponding value σ'_{a2} of Equation 2 average 4% below, as opposed to the σ'_{a3} of Equation 3, about 1% below.
- Even on the dried crust, there is a good agreement between the σ'_a of the Equations 1 and 2 and those of the consolidation tests.
- The parameters t_2 and t_3 remain practically constant, as shown in Figures 3b and 3c; t_1 presented with small deviations (Figure 3a).

8.2 Torp clay, Sweden

Torp Clay, which occurs in Munkedal, Sweden, in the valley of the Örekilsälven River, was deposited in the post-glacial period, after around 9,000 years BP. At the top there is a sandy layer, followed by clay, with silt and sand lenses, and, at greater depths, layers of silt and sand (Larsson & Åhnberg, 2003 and 2005). Originally, the clay was normally or only slightly overconsolidated. But it became overconsolidated due to erosive processes, landslides, and excavations in the 1980's; in summary, because of recent real unloading. But these authors proposed for σ'_a expression:

$$\sigma'_a = 1.15 \cdot \left(\sigma'_{vo} + 100\right) \tag{31}$$

even though creep effects could not be inferred from the test results (page 88 of Larsson & Åhnberg 2003). In view of this statement, r=1 is a working hypothesis in the present paper.

Unlike Bothkennar clay, Torp Clay presents a certain heterogeneity, which is manifested, for example, in the natural specific unit weights (γ_n) of good quality undisturbed samples, varying roughly between 16.5 and 20.0 kN/m^3, as shown in Figure 4.

Larsson and Åhnberg presented results of CPTUs done in various sections in the Torp area. Of interest to the present study is the CPTU-S9 (Figure 5), executed in an excavated area, with water level at a depth of 2m, and which was the object of studies by Mayne (2017) and Ferreira (2021), already mentioned. The values of q_t and u_2 vary linearly with depth, as shown in Figure 5, and the 3 Mayne's Formulas, i.e., Equations 1, 2 and 3, can be used again, with the corresponding empirical factors summarized in Table 1, with r=1.0, as admitted above.

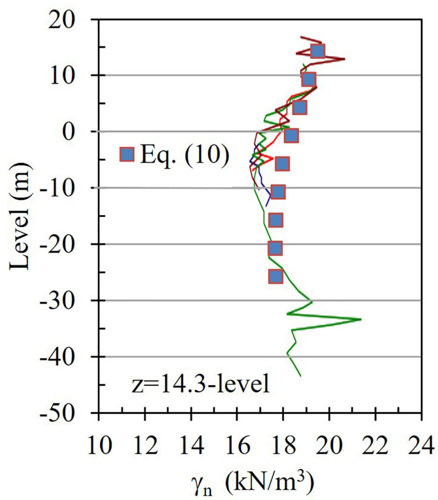

Figure 4. Laboratory Specific Unit Weight – Torp Clay - Prepared with data from Larsson & Åhnberg (2003).

The values of the empirical factors ($N_{\sigma t}$, k_2, k_3) are presented in Table 4. Figure 6 shows how σ'_a and the *OCR* vary in depth.

The parameters t'_1, t'_2 and t'_3 were determined through Equations (22), (23) and (24), using the values of *b* and *d* from Figure 5a and 5b, respectively, and γ_n=18,27 kN/m3 (average value of $\overline{\gamma}_n$), with results presented in Table 5 and inserted into Figure 7 (dashed lines). This figure shows the variations of t_1, t_2 and t_3 with depth, using the same Equations 22, 23 and 24 and the values of q_t, σ_{vo}, u_2 and u_o, presented in Figure 5.

The average values of t_1/t'_1, t_2/t'_2 and t_3/t'_3 are 1.20, 1.20 and ~1.00, respectively, which signifies, from Equations 25, 26 and 27, mean deviations of 20% between σ'_{a1} and σ'_{a2}; also 20% between σ'_{a1} and σ'_{a3} and ~0% between σ'_{a2} and σ'_{a3}.

Supposing again that γ_n was not available, the attempt to directly solve Equations 18-a would lead to the following terms of Equation 18-b:

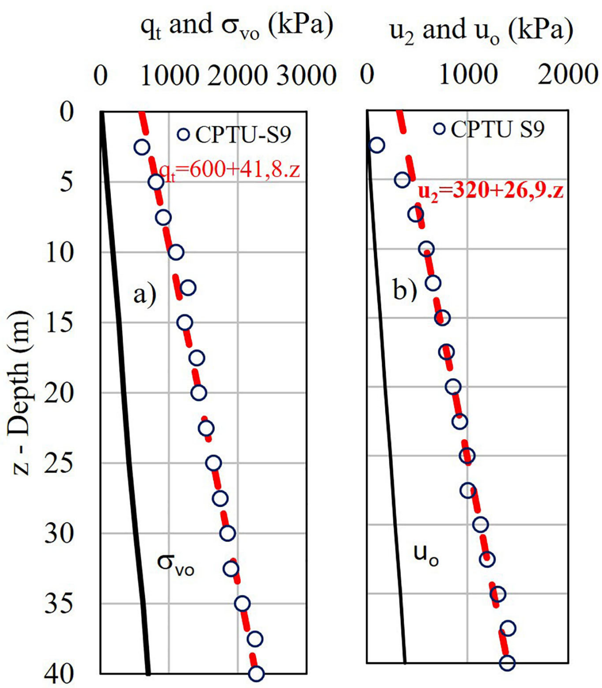

Figure 5. CPTU-S9 – Torp Clay (Sweden). Prepared with data from Mayne (2017): a) q_t and σ_{vo}; and b) u_2 and u_o.

Table 4. Results obtained for the 3 empirical factors of Equations 1, 2 and 3 - Torp Clay.

Item	$N_{\sigma t}$	k_2	k_3
Mayne (2017)	3.0	0.60	0.54
Calculated	2.35 to 3.15 Average=2,87	0.51 to 0.64 Average=0.56	0.45 to 0.56 Average=0.49

Table 5. Values of t'_1, t'_2 and 't_3 obtained through Equations 22, 23 and 24 as a function of *b*, *d* and γ_n.

Item	t'_1	t'_2	t_3'
Bothkennar Clay	1.58	1.39	0.88

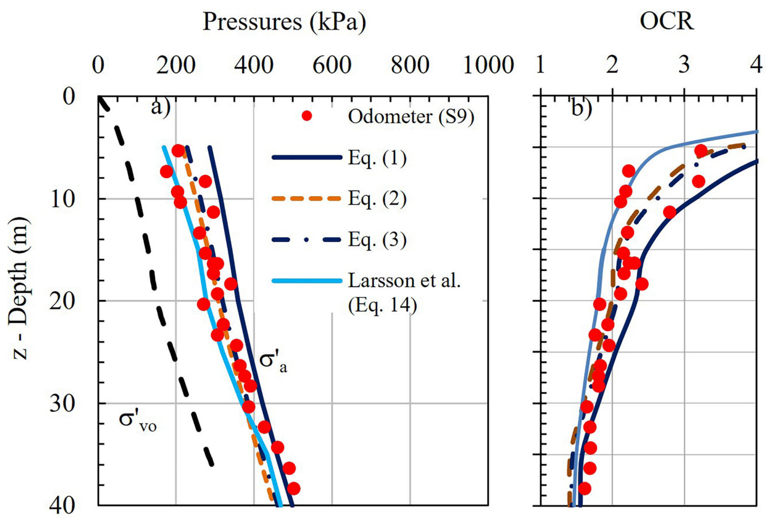

Figure 6. a) Effective Pressures σ'_{vo} and σ'_{a}; and b) OCR – Torp Clay (Sweden).

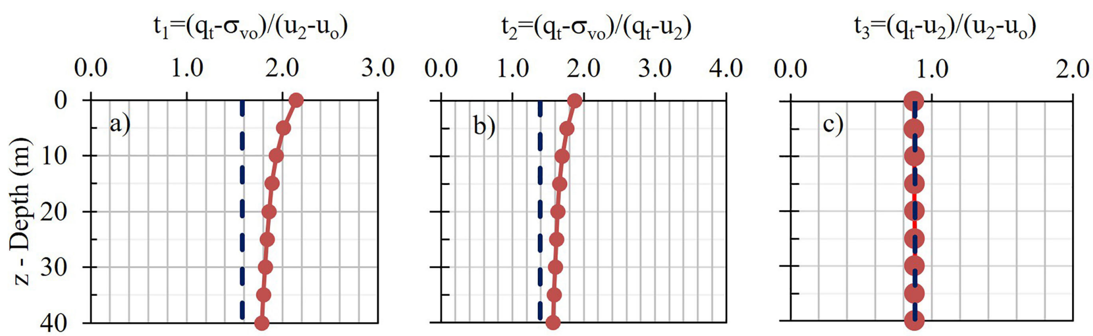

Figure 7. Values of: a) t_1; b) t_2 and c) t_3 varying with depth. The dashed lines represent t'_1, t'_2 and t'_3 – Torp Clay (SW).

$$T = \begin{bmatrix} 1.148 & -2.148 & 1 \\ 1 & -1.886 & -1 \\ 1 & -1.878 & 0 \end{bmatrix},$$

$$U = \begin{bmatrix} 41.8 \\ 26.9 \\ 9.9 \end{bmatrix} and \qquad (32)$$

$$N = \begin{bmatrix} 0.00 \\ -18.86 \\ -8.78 \end{bmatrix}$$

Again, multiplying T by U results in a vector whose transpose is [0.00 -18.86 -8.78}, which is equal to N. It is worth mentioning that the determinant of the T is 9.2E-06, almost zero, confirming that the system of Equations 32 is ill conditioned. This solution gave the unrealistic γ_n =9.9 kN/m^3 (third element of U) and so was discarded. The corresponding values of yield pressures were also unrealistic.

From these results it can be concluded that:

- The working hypothesis that r=1 was validated.
- The 3 Mayne's Formulas, i.e., Equations 1, 2 and 3, with the empirical factors of Table 1, can be used even for soils with a certain heterogeneity, with specific unit weights varying with depth as shown in Figure 4; hence the shapes of the yield pressures in Figure 6a have inflections and are curvilinear.
- This heterogeneity affected the empirical factors, as shown in Table 4; in average terms, they deviate from the Mayne's values by -4% for $N_{\sigma t}$, -7% for k_2 and -12% for k_3.
- The σ'_{a2} and σ'_{a3} calculated using Equations 2 and 3 align more closely with the oedometer results than σ'_{a1} from Equation 1, which showed a 20% deviation from the others.
- The OCR (Over Consolidation Ratio) of Torp Clay ranges from 4.0 to 1.4.
- The parameter t_3 remains practically constant and equals t'_3, as shown in Figure 7c, confirming the match between σ'_{a2} and σ'_{a3}.

8.3 Jurubatuba clay, Santos, Brazil

The clay of Jurubatuba is Holocene soft clay, deposited as a Fluvio-Lacustrine Sediment in Santos Coastal Plain, São Paulo, Brazil, after the last sea level transgression, which occurred 8,000 years BP. In their natural state, the yield stress difference (σ'_a-σ'_{vo}) ranges from 20 to 30 kPa, due to negative sea level oscillations (Massad, 2009). It stands today under 3.5m height of a man-made fil, which implies r=1 and OCR=1.

Figure 8 presents the results of a CPTu carried out in the site. The water level was at a depth of 1.8 m and the average natural specific unit weight of the clay (γ_n) was not available. For similar soils in the area, γ_n ranges from13.5 to 16.5 kN/m^3. Again, a linear variation of q_t and u_2 with depth (z) is observed, as shown in Figure 8.

As γ_n was unknown, the iterative procedure presented in item 6 was used. It was initiated taking b=30 kPa/m, as shown in Figure 8a, and γ_n=13 kN/m^3 (assumed). After the iterations, γ_n=14.2 kN/m^3 resulted, with d equals to 20.9 kPa/m, very close to the value indicated in Figure 8b. Other results are presented in Tables 6 and 7 and in Figure 9. It is worth mentioning that during iteration, t_1 and t_2 varied with γ_n, but t_3 remained constant.

The yield pressures σ'_{a1}, σ'_{a2} and σ'_{a3}, computed by the 3 Mayne's Formulas, i.e., Equations 1, 2 and 3, were the same, as shown in Figure 10, because the relations t_1/t'_1, t_2/t'_2 and t_3/t'_3 are equal to 1.

In this case, the terms of Equation 18-b are:

$$T = \begin{bmatrix} 0.728 & -1.728 & 1 \\ 1 & -1.454 & -1 \\ 1 & -1.844 & 0 \end{bmatrix},$$

$$U = \begin{bmatrix} 30.0 \\ 20.8 \\ 14.2 \end{bmatrix} \textit{and} \qquad (33)$$

$$N = \begin{bmatrix} 0.00 \\ -14.54 \\ -8.44 \end{bmatrix}$$

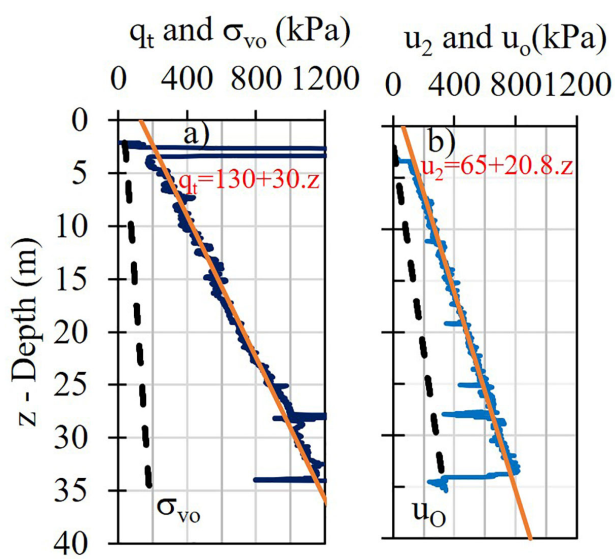

Figure 8. CPTU in Jurubatuba Clay, Santos, Brazil: a) q_t and σ_{vo}; and b) u_2 and u_o.

Table 6. Results obtained for the 3 empirical factors of Equations 1, 2 and 3 - Jurubatuba Clay.

Item	$N_{\sigma t}$	k_2	k_3
Mayne (2017)	3.0	0.60	0.54
Calculated	3.7	0.46	0.39

Table 7. Values of t'_1, t'_2 and t'_3 obtained through Equations 15, 16 and 17 as a function of b, d and γ_n.

Item	t'_1	t'_2	t'_3
Jurubatuba Clay	1.71	1.46	0.85

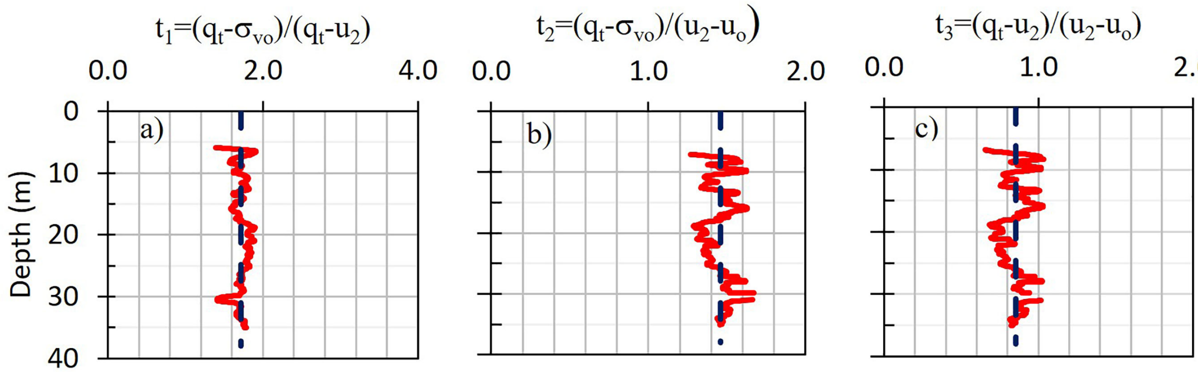

Figure 9. Values of: a) t_1; b) t_2 and c) t_3 varying with depth. The dashed lines represent t'_1, t'_2 and t'_3 – Jurubatuba Clay, Santos, Brazil.

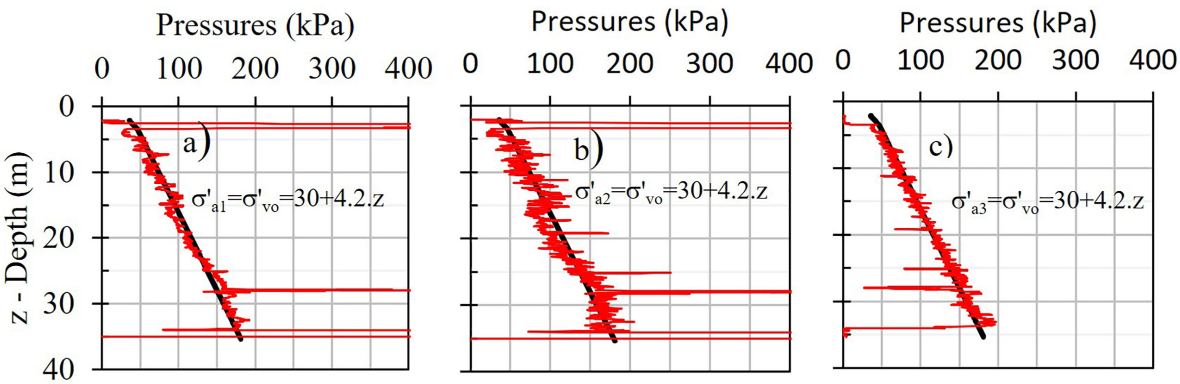

Figure 10. Pressures– Jurubatuba Clay (Santos, Brazil): a) σ'_{a1}; b) σ'_{a2} and c) σ'_{a3}.

Note that multiplying T by U results a vector whose transpose is [0.02 -14.56 -8.47}, which differs from N by a figure of 0.02. It is worth mentioning that the determinant of T is 4.5E-03, almost zero, confirming that the system of Equations 18-a is ill conditioned. The third element of U displayed γ_n=14.2 kN/m3, equals to the above iterated value, and which is in the range from13.5 to 16.5 kN/m^3 of similar soils in Santos.

From these results it can be concluded that:

- The clay is normally consolidated (OCR=1), as stated before, due to the recent 3.5m man-made fill.
- The value of γ_n, determined iteratively, as proposed in item 6, resulted in 14.2 kN/m^3, into the expected range of similar soils in Santos Coastal Plain.
- The yield pressure determined by the 3 Mayne's Formulas gave practically the same result (Figure 10), confirming the OCR value of 1 (clay normally consolidated by the man-made fill).
- But the computed empirical factors $N_{\sigma t}$, k_2 and k_3 (Table 6) differ from those indicated by Mayne (2017). The differences are +23% for $N_{\sigma t}$, -23% for k_2 and -28% for k_3.
- The average values of t_1, t_2 and t_3 coincide with the corresponding values of t'_1, t'_2 and t'_3 (Figure 9).

9. Conclusions

The yield pressures can be determined based on high quality piezocone tests (CPTU), using the 3 formulas indicated by Mayne, i.e., Equations 1, 2 and 3, provided that their empirical factors are adjusted, considering both the geological history of the marine soils and the linearity between q_t and u_2 with depth. These conditions define the range of applicability of the proposed method, together with the natural specific unit weights (γ_n) of good quality undisturbed samples.

The necessary condition to obtain identical or consistent yield pressures using the 3 Mayne's Formulas has been established, although it depends on solving ill conditioned systems of linear equations. To address this challenge, an iterative procedure was introduced to enforce consistency, relying primarily on the known rates of increase of q_t and u_2 with depth. But geotechnical judgments are required for acceptance of the results, as shown in the analyzed case histories. For Bothkennar Clay and Torp Clay deviations occurred probably due to imprecisions in piezocone measurements and rounding errors during calculations, because the systems of linear equations are sensitive to such inaccuracies. A method was set up to evaluate and quantify these deviations.

For Bothkennar Clay, a very good fitting was observed between the calculated yield pressures and those obtained through oedometer tests; the deviations among the calculated values were less than 4%.

For Torp Clay the yield pressures calculated using Equations 2 and 3 aligned closely with the oedometer results (deviations close to zero) than those obtained from Equation 1, which showed a 20% deviation from the others.

For the Jurubatuba Clay it was possible to solve the system of linear equations and determine the specific unit weight and yield pressure, consistent with the geological and geotechnical knowledge of the region and the case. The iterative approach resulted in excellent agreement among Mayne's Formulas, with negligible differences between the calculated values.

Finally, the proposed empirical factors may be applicable even to heterogeneous marine clays with some variation in their specific unit weights, as was the Torp Clay case.

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Declaration of interest

The authors have no conflicts of interest to declare and there is no financial interest to report.

Data availability

The datasets generated analyzed in the course of the current study are available from the corresponding author upon request.

Declaration of use of generative artificial intelligence

This work was prepared without the assistance of any generative artificial intelligence (GenAI) tools or services. All aspects of the manuscript were developed solely by the authors, who take full responsibility for the content of this publication.

List of symbols

$a; b$	Constants of Equation 5
$c; d$	Constants of Equation 6
$k_2; k_3$	Empirical factors of Equations 2 and 3
q_t	Corrected Cone Point Resistance
r	Term of Equation 4-b
t	Aging time
$t_i; t'_i$	Auxiliary variables (i=1, 2 and 3)
t_p	Time for primary consolidation to occur
u_o	Hydrostatic pressure
u_2	Poro-pressure at the cone base
$z; z_{wl}$	Depth; depth of water level
Bq	Pore pressure parameter
BP	Before Present
$C_c; C_r$	Compression and recompression indexes
CPTU	Cone Penetration Test
$C_{\alpha e}$	Coefficient of secondary consolidation
N	Vector
$N_{\sigma t}$	Empirical factor of Equation 1
OCR	Over Consolidation Ratio
T	Matrix 3 x 3
U	Vector of the variables b, d and γ_n
γ_o	Specific weight of water
$\gamma_n; \gamma'$	Natural and effective specific unit weights of soil
$\overline{\gamma}_n$	Weighted average of γ_n (Equation 10)
σ'_a	Yield pressure
σ'_{a1}	Yield pressure from Equation 1
σ'_{a2}	Yield pressure from Equation 2
σ'_{a3}	Yield pressure from Equation 3
σ_{vo}	Initial Total Vertical Pressure
σ'_{vo}	Initial Effective Vertical Pressure
Δp	Overload

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